Developing an Optimal Sensing Strategy for Accurate Freeway Travel Time Estimation and Traveler Information

Robert L. Bertini
Department of Civil and Environmental Engineering
Portland State University
P.O. Box 751
Portland, OR, 97207 USA
Email: bertini@pdx.edu
Phone: 503-725-4249
Fax: 503-725-5950

David J. Lovell*  
Department of Civil and Environmental Engineering and Institute for Systems Research
University of Maryland
1173 Glenn Martin Hall
College Park, MD  20742 USA  
Email: lovell@eng.umd.edu
Phone: 301-405-7995
Fax: 301-405-2585

* Corresponding author

Submitted for presentation and publication to the
87th Annual Meeting of the Transportation Research Board
January 13–17, 2008

Revised November 4, 2007

TRB 2008 Annual Meeting CD-ROM  Paper revised from original submittal.
Abstract. Accurate freeway travel time estimates are critical for transportation management and traveler information—both infrastructure-based and in-vehicle. Infrastructure managers are interested in estimating optimal freeway sensor density for new construction and retrofits. This paper describes a concept developed from first principles of traffic flow for establishing optimal sensor density based on the magnitude of under-and over-prediction of travel time during shock passages when using the midpoint method. A suggested aggregate measure developed from vehicle hours traveled (VHT) is described for a reasonable range of detector densities. Extensions of the method to account for both recurring and nonrecurring congestion are included. Finally some suggestions for future research are described.

INTRODUCTION

Accurate travel time estimation is important for transportation situational awareness and management. In the U.S., freeways account for 3% of the national highway mileage, but accommodate more than 30% of the vehicle-miles traveled (VMT). The alleviation of congestion on urban freeways is receiving heightened attention, and transportation agencies are applying improved management strategies to reduce congestion and improve travel time reliability. Even to the extent that congestion cannot be reduced markedly, there is benefit in accurate predictions of travel time that enable better informed traveler decisions. Incident management and traveler information systems can be implemented at relatively low cost. However, such systems rely upon accurate measurement of traffic parameters such as flow, speed, travel time, and delay. Usually these data are measured by fixed sensors (loop detectors, video cameras, radar sensors, etc.) or by mobile data sources such as automatic vehicle identification (AVI) toll tags or automatic vehicle location (AVL) probe vehicles and in the future by such technologies as vehicular ad hoc networks. In order to answer the question “how much detection is needed,” this paper focuses on the common use of fixed sensors (such as loop detectors) as a basis for formulating an optimal detector placement strategy.

Figure 1 illustrates a hypothetical space-time (x-t) plane of length λ and time interval t. A set of vehicle trajectories is shown (in grey) with that of vehicle i being highlighted in black. If equipped with AVL or any other data logging system, vehicle i’s trajectory could be plotted and all information necessary to completely describe its path would be known, including its actual travel time and its speed (slope of trajectory) at any point. If one assumes a free flow speed, the free flow travel time can be computed and the delay (actual minus free flow travel time) for vehicle i is known. In practice it is more common to use sensors at fixed points (such as point x_i) to measure speed and subsequently extrapolate that speed over a segment. In Figure 1 one can observe how a speed measured at x_i is

![FIGURE 1 Segment travel time features.](image-url)
extrapolated over a segment of length $\lambda$ resulting in the calculation of the extrapolated travel time. As shown, the estimate does not perfectly match the actual travel time. The magnitude of this difference, as a function of detector density, is the topic of interest in this paper. Ignoring sensing errors, the passage of a shock represents the “worst case scenario” for travel time prediction, so the methods presented in this paper can be interpreted as a form of robust decision analysis about sensor spacing.

**TRAVEL TIME ESTIMATION FRAMEWORK**

A fundamental traffic flow relation is assumed, as shown in Figure 2. In particular, the triangular shape is used – this is common in the literature when broad questions need to be addressed with a minimum of complications. A congested state C (flow $q_c$ and speed $v_c$) and uncongested states A, B, and D (flows $q_A$, $q_B$, $q_D$, speed $v_f$) are shown. Below the flow-density ($q$-$k$) diagram is an $x$-$t$ plane showing a bottleneck (either recurrent or nonrecurrent) at location $bn$. We will assume, for the sake of simplicity, that the bottleneck state is binary; i.e., it is either active, with some reduced capacity captured by state C, or it is inactive, in which case the nominal roadway capacity can be permitted. Following the rules of standard first-order macroscopic traffic dynamics, and assuming the nominal traffic state was A, there is a transition between uncongested state A and congested state C, defined by a shock of velocity $v_{AC}$. Figure 2 shows that for an arbitrary highway segment (separated by the two dashed lines) transition AC is bounded by a rectangle as the shock passes. If this were an active bottleneck that was deactivated at time $t_{deact}$, then transition CD would occur, from congested state C to uncongested state D, marked by a backward-moving recovery wave of velocity $v_{CD}$. Transition DA (the return to nominal conditions) is separated by a forward-moving recovery wave of velocity $v_f$. We do not assume that these three transitions always occur in this configuration or order, and the computations later in the paper assess each type of state transition separately. The overall pattern of Figure 2 is simply convenient because all of the interesting possible transitions are shown.

![Diagram of traffic flow relation and dynamics](image)

**FIGURE 2** Assumed traffic flow relation and traffic dynamics.

The first-order hydrodynamic model approximates actual vehicle trajectories, such as those illustrated in Figure 1, with piecewise-linear trajectories. For the purposes of travel time measurements over a link, this is irrelevant, because we measure one vehicle’s travel time as the horizontal difference between the endpoints of its trajectory on the link, and the microscopic details of the shapes of speed transitions in the vicinity of a shock do not matter.
Figure 3 shows an $x$-$t$ plane for an actual freeway corridor, with the $y$-axis as distance and the $x$-axis as time (4:00 to 20:00). The figure shows speed as a “color,” for a 23-mile corridor (northbound I-5 in Portland, Oregon) on Feb 8, 2007. Mileposts are shown on the left $y$-axis, and the locations of five variable message signs (VMS) are overlaid with their milepost locations shown on the right $y$-axis. Downtown Portland is located at approximately milepost 300. There are more than 500 lane and ramp sensors on the Portland area freeways, at about 138 locations, with an average sensor spacing of 1.24 miles. Nearly all of Portland’s sensors are located just upstream of on-ramps, as part of the region’s ramp metering system. The Oregon Department of Transportation (ODOT) displays travel time information to Downtown on VMS 1, 2 and 3 during the AM peak period. ODOT and others are interested in increasing sensor density toward some “optimal” value, in order to improve the accuracy of travel time information in a cost-effective manner.

For the section between milepost 294 and 295, one can observe state A (uncongested) from 4:00 to 7:00. This is followed by transition AC (uncongested to congested) marked by the upstream propagation of a queue through the section. A line with slope $v_{AC}$ has been superimposed on the figure. Next, the section is in state C (congested) until after 8:30. This is followed by transition CD (congested to uncongested), marked by the passage of a backward moving recovery wave. A line with slope $v_{CD}$ has been superimposed on the figure. Traffic remains in state D until after 9:00 when a forward moving wave (at speed $v_f$) marking transition DA passes through the section. The remainder of the day was marked by uncongested conditions in the section (e.g. state(s) similar to A). This pattern can be typical for both recurrent and nonrecurrent congestion. In the case of the 1 mi section mentioned here, these transitions occurred only once. However, looking further downstream at another next section (for example between milepost 299 and 300), one can observe that there were two such transitions on this particular day.
CALCULATING TRAVEL TIME DURING STATIONARY CONDITIONS

Figure 4 illustrates that travel times during regimes A and D (fully uncongested), C (fully congested), and during transition DA (uncongested) can be estimated by:

\[ t_{t_f} = \frac{1}{v_f}, \quad t_{t_c} = \frac{1}{v_c} \]

where \( t_{t_f} \) is the free flow travel time and \( t_{t_c} \) is the congested travel time. If the sensor where speed is measured is located at the center of the segment, this is called the midpoint method. Travel time estimation over a segment can be performed accurately if the traffic state within the segment is either fully uncongested or fully congested. In addition, \( t_{t_c} \) is an upper bound on the actual travel time through the segment and \( t_{t_f} \) is a lower bound on the actual travel time. Empirical studies have shown that travel time estimation during stationary conditions (either uncongested or congested) is usually quite accurate (1, 2). For simplicity, in this paper, the midpoint method will be used; however, other methods have been developed for improving travel time estimates (3, 4) and can be studied further later.

TRAVEL TIME ESTIMATION DURING TRANSITIONS

As initially illustrated in Figure 3, there are two basic transition types in freeway traffic flow: uncongested to congested (e.g. AC) and the reverse (e.g. CD). These transitions can occur multiple times in a given section as queues propagate and dissipate and sometimes combine with one another. Queues may be caused by recurrent bottlenecks or by incidents. These transient conditions induce errors in travel time estimations based on current measured conditions, and depending on the specifics, can result in either overprediction or underprediction. We will treat underprediction as more problematic than overprediction, since travelers whose travel times are much longer than predicted at their entry to a segment will be more likely to be dissatisfied. The method involves choosing a sensor spacing as a result of a tradeoff between over- and under-prediction and for the reasons cited above, underprediction will be given a higher weight in this tradeoff. A hypothetical segment of length \( \lambda = 1 \) mi will be considered for comparison purposes. For the sake of numerical examples a range of five sensor spacings \( s \) (0.1 to 1 mi) will be used and assumed traffic flow parameters will be: \( q_A = 2000 \) vph, \( q_C = 1800 \) vph, \( v_f = 60 \) mph, \( v_c = 30 \) mph, \( v_{CD} = -17.1 \) mph, and \( v_{AC} = -7.5 \) mph. For the calculations that follow, we will consider as the domain of the travel time computations a region of the time-space plane that is bounded by the link endpoints, and includes exactly those vehicles whose trajectories were affected by the passage of the shock.

UNDERPREDICTING TRAVEL TIME DURING REGIME AC

Figure 5 illustrates transition AC from uncongested conditions with vehicles traveling at \( v_f \) to a congested state with vehicles traveling at \( v_c \). The figure illustrates a backward moving shock passing through the segment of length \( \lambda = 1 \) mi, and sensor spacing \( s \), at a speed \( v_{AC} \). Vehicle \( j_1 \) is the last vehicle to have driven at speed \( v_f \) throughout the section, and vehicle \( j_3 \) is the first vehicle to traverse the section at consistent speed \( v_c \). All vehicles between \( j_1 \) and \( j_3 \) change speeds at some point in the section, and their average speeds are therefore between \( v_f \) and \( v_c \). The dark shading in the figure encapsulates exactly these vehicles. The figure also shows that the sensor continues to record...
speed $v_f$ until time $t_c$, a lag time $\alpha = -s/2v_{AC}$ after the shock enters the section. After vehicle $j_1$ and until $t_c$, drivers expect a free flow trip time through the entire section while their actual trip time will be higher. For example, driver $j_2$ enters the section an instant before time $t_c$ and expects a speed of $v_f$ through the entire section, but experiences a longer actual travel time $z$. For vehicles after $j_1$ entering the section before time $t_c$, travel time is underpredicted by an amount equal to the difference between the expected travel time (dashed trajectory in the figure) and the actual travel time (solid trajectory). Prior to encountering the shock, vehicle $j_2$ travels a distance

$$\frac{v_f \left( \frac{l - s}{2} \right)}{v_f - v_{AC}}$$

at speed $v_f$, and drives the remainder of the link at speed $v_c$. Thus, the travel time experienced by vehicle $j_2$ is:

$$z = \frac{v_f \left( \frac{l - s}{2} \right)}{v_f - v_{AC}} + \frac{v_f \left( \frac{l - s}{2} \right)}{v_f - v_{AC}} = \frac{l - s}{2} + \frac{l \left( v_f - v_{AC} \right) - v_f \left( \frac{l - s}{2} \right)}{v_f \left( v_f - v_{AC} \right)}$$

$$= \frac{lv_f - v_f \left( \frac{s}{2} \right) + lv_f - lv_{AC} - lv_f + v_f \left( \frac{s}{2} \right)}{v_f \left( v_f - v_{AC} \right)} = \frac{l \left( v_f - v_{AC} \right) + \frac{1}{2}s \left( v_f - v_f \right)}{v_f \left( v_f - v_{AC} \right)}$$

The amount by which travel time is underpredicted is maximal for this vehicle, and this maximum error is given by:

$$u_{\text{max}} = z - tt_f = \frac{l \left( v_f - v_{AC} \right) + \frac{1}{2}s \left( v_f - v_f \right)}{v_f \left( v_f - v_{AC} \right)} - \frac{l}{v_f}$$

For example, if $\lambda = s = 1$, the driver of vehicle $j_2$ expects a travel time of 1 min, but actually experiences a 1.56 min trip, which is more than 50% longer than expected. If one assigns zero weight to any travel time overprediction (between time $t_c$ and the time the shock reaches the upstream end of the section), for now one can neglect any flow entering the section after time $t_c$. The remainder of this section only considers this scenario, since it is assumed that traffic management officials want to avoid giving drivers false expectations of low travel times, when they actually experience longer ones. One can quantify the predicted and actual vehicle-hours traveled (VHT$^{\text{pred}}$ and VHT$^{\text{act}}$) solely for vehicles experiencing underprediction (superscript $u$) over the hypothetical segment. To do so, we count the number of vehicles in each situation, and multiply by the average travel time (either predicted or actual). Since

FIGURE 5 Travel time estimation during regime AC.
the trajectories and shocks are all linear, the average travel time for a group of vehicles with a similar disposition will always be the midpoint of the best and worst travel times amongst that set.

The trajectories of those vehicles whose travel time is underpredicted cross the upstream end of the link over a time span of

$$t_f + \alpha = \frac{l}{v_f} - \frac{s}{2v_{AC}}$$  (3)

At this location, the flow is $q_A$; hence the number of vehicles in this condition is

$$q_A \left( \frac{l}{v_f} - \frac{s}{2v_{AC}} \right)$$  (4)

Each of these vehicles was expected to travel at free flow speed and therefore have free flow travel time across the link, so the predicted VHT for vehicles whose travel time was underestimated in the AC transition is:

$$VHT_{pred} = \frac{q_A l}{v_f} \left( \frac{l}{v_f} - \frac{s}{2v_{AC}} \right)$$  (5)

The actual total travel time, as mentioned above, can be found by multiplying the same number of vehicles by their expected travel time, which is midway between the highest and lowest travel times for this group of vehicles. The lowest travel time is the free-flow travel time, $tt_{f}$. The vehicle with the highest travel time is trajectory $j_2$ of Figure 5, whose travel time was computed in equation (1). The average travel time for all vehicles is then the average between these lowest and highest values, and the total actual travel time for these vehicles, whose travel time was underpredicted, can be found by multiplying this average by the number of vehicles in equation (4), resulting in the following:

$$VHT_{act} = \frac{q_A}{2} \left( \frac{l}{v_f} - \frac{s}{2v_{AC}} \right) \left( \frac{l}{v_f} + \frac{l(v_e - v_{AC}) + 1/2s(v_f - v_e)}{v_e(v_f - v_{AC})} \right)$$  (6)

For $\lambda = s = 1$, the predicted VHT/mile is 2.78 veh-hr/mile, yet the actual VHT/mile is 3.55 veh-hr/mile, a 22% error over the collection of vehicles entering transition AC before $t_c$. Vehicles experience actual underpredictions between 0 and 0.56 min.

In order to extend these calculations to arrangements with greater sensor density, Figure 6 illustrates how predicted and actual VHT will change with increased sensor density. The increased sensor placement reduces the lag time $\alpha$ so vehicles entering the segment receive the congestion message sooner. This reduces the magnitude of the...
VHT composed of travel time underprediction, and the figure indicates this using the darker shaded areas. One can see that as the lag time decreases, the VHT of traffic impacted by travel time underprediction will decrease in this 1 mi section. For now the issue of overprediction is set aside.

Table 1 shows the values of $u_{\text{max}}$ and lag time $\alpha$ for regime AC as a function of detector spacing $s$. The table also shows the predicted and actual VHT/mile for a range of $s$. To reiterate, for this hypothetical 1 mi segment, vehicles entering the section prior to $t_c$ will expect free flow conditions, but will instead experience progressively longer travel times (the lag time gets shorter with increased detector density). For this set of vehicles, the actual VHT is higher than the predicted VHT. Even with sensors at 0.1 mi spacing, the VHT error is 7% and vehicles expecting a 1 min travel time can actually experience 1.16 min, a 16% underprediction. The gap between the predicted and actual VHT grows with larger sensor spacing. For the range of sensor spacing considered, the VHT error falls between 7% and 22%. For the average ODOT sensor spacing of 1.2 mi, the VHT error is 24%.

Here, more detection is better if the goal is to minimize underprediction. Spacing between 0.25 and 0.50 mi would keep $u_{\text{max}}$ below 33% of $t_f$. Table 1 also includes a sixth row with $s = 0$, which is equivalent to some form of ubiquitous sensor coverage.

**Overpredicting Travel Time During Regime AC**

The previous section examined the impact of only the travel time underprediction during the transition from uncongested to congested conditions on a freeway section (regime AC). Travel times for vehicles entering the section after time $t_c$ and until the shock crosses the upstream end of the link are overpredicted. The duration over which these vehicles cross the upstream end of the link is

$$\frac{1}{v_{AC}} \left( s - l \right),$$

and they do so at flow rate $q_A$. The number of such vehicles is therefore

$$\frac{q_A}{v_{AC}} \left( s - l \right),$$

and each expected the congested travel time $l/v_c$.

For the transition regime AC, including only the overprediction, the predicted VHT (with superscript $o$) is:

$$VHT_{\text{pred}}^o = \frac{q_A}{v_{AC}} \left( s - l \right).$$

The actual VHT for these vehicles can be computed in a manner identical to equation (6), except that the number of vehicles in this condition is given by (7) and the two extreme travel times whose average is the average for all vehicles are $l/v_c$ and the travel time for vehicle $j_2$ as shown in equation (1). This results in the following total travel time for vehicles in transition regime AC, whose travel time was overpredicted:

$$VHT_{\text{act}}^o = \frac{q_A}{2v_{AC}} \left( s - l \right) \frac{1 - \frac{l(v_c - v_{AC}) + \frac{1}{2}s(v_f - v_{AC})}{v_c(v_f - v_{AC})}}{1}.$$

Table 1 shows that for $\lambda = s = 1$, for vehicles entering the section after $t_c$ the predicted VHT is 4.44 veh-hr, and the actual VHT is 3.95 veh-hr, an overprediction of 13%. Most drivers would be pleasantly surprised by the shorter travel time but it is not safe to assume that over-prediction is always benign. In a dynamic route choice application, for example, it could disguise a better link and thus lead to suboptimal route recommendations. With increased sensor density, the percent error in VHT increases to a maximum of 27% overprediction for $s = 0.10$.

If one assumes that under- and overpredicted VHT can be added together numerically (with one canceling out the other), the total predicted VHT in regime AC would be $VHT_{\text{pred}} + VHT_{\text{pred}}^o$, and the actual VHT in regime AC would be $VHT_{\text{act}} + VHT_{\text{act}}^o$ (7.50 veh-hr). Here one time unit of underprediction is canceled out by a time unit overprediction. As shown in Table 1, when the two components are added, for $\lambda = s = 1$, the effect is still a 4% underprediction in VHT. For decreasing values of $s$, the aggregate effect is overprediction, up to 23% with $s = 0.10$. Figure 7 illustrates this graphically, where the optimal sensor spacing might be close to 0.8 mi, which is where the VHT error line crosses from overall underprediction to overprediction. For the situation where drivers might assign a higher “price” to underprediction than to overprediction, Figure 7 shows, with the line series labeled “penalty,” the
TABLE 1 Segment Travel Time Statistics During Regimes AC and CD

Regime AC

<table>
<thead>
<tr>
<th>s</th>
<th>u_{max}</th>
<th>Lag Time (min)</th>
<th>VHT/mile</th>
<th>Under % Error</th>
<th>VHT/mile</th>
<th>Over % Error</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.56</td>
<td>4.00</td>
<td>2.78</td>
<td>22%</td>
<td>4.44</td>
<td>-13%</td>
<td>4%</td>
</tr>
<tr>
<td>0.50</td>
<td>0.33</td>
<td>2.00</td>
<td>1.67</td>
<td>14%</td>
<td>6.66</td>
<td>-20%</td>
<td>-11%</td>
</tr>
<tr>
<td>0.33</td>
<td>0.26</td>
<td>1.33</td>
<td>1.30</td>
<td>11%</td>
<td>7.40</td>
<td>-23%</td>
<td>-16%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.22</td>
<td>1.00</td>
<td>1.11</td>
<td>10%</td>
<td>7.77</td>
<td>-24%</td>
<td>-19%</td>
</tr>
<tr>
<td>0.10</td>
<td>0.16</td>
<td>0.40</td>
<td>0.78</td>
<td>7%</td>
<td>8.44</td>
<td>-27%</td>
<td>-23%</td>
</tr>
<tr>
<td>0</td>
<td>0.11</td>
<td>0.00</td>
<td>0.56</td>
<td>5%</td>
<td>8.88</td>
<td>-29%</td>
<td>-26%</td>
</tr>
</tbody>
</table>

Regime CD

<table>
<thead>
<tr>
<th>s</th>
<th>u_{max}</th>
<th>Lag Time (min)</th>
<th>VHT/mile</th>
<th>Under % Error</th>
<th>VHT/mile</th>
<th>Over % Error</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.56</td>
<td>4.00</td>
<td>0.88</td>
<td>14%</td>
<td>3.75</td>
<td>-21%</td>
<td>-12%</td>
</tr>
<tr>
<td>0.50</td>
<td>0.33</td>
<td>2.00</td>
<td>1.32</td>
<td>19%</td>
<td>2.88</td>
<td>-15%</td>
<td>-1%</td>
</tr>
<tr>
<td>0.33</td>
<td>0.26</td>
<td>1.33</td>
<td>1.46</td>
<td>21%</td>
<td>2.58</td>
<td>-13%</td>
<td>2%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.22</td>
<td>1.00</td>
<td>1.54</td>
<td>22%</td>
<td>2.44</td>
<td>-12%</td>
<td>4%</td>
</tr>
<tr>
<td>0.10</td>
<td>0.16</td>
<td>0.40</td>
<td>1.67</td>
<td>23%</td>
<td>2.18</td>
<td>-11%</td>
<td>7%</td>
</tr>
<tr>
<td>0</td>
<td>0.11</td>
<td>0.00</td>
<td>1.75</td>
<td>24%</td>
<td>2.00</td>
<td>-10%</td>
<td>9%</td>
</tr>
</tbody>
</table>

Regimes AC and CD

<table>
<thead>
<tr>
<th>s</th>
<th>u_{max}</th>
<th>Lag Time (min)</th>
<th>VHT/mile</th>
<th>Under % Error</th>
<th>VHT/mile</th>
<th>Over % Error</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.56</td>
<td>4.00</td>
<td>2.78</td>
<td>22%</td>
<td>4.44</td>
<td>-13%</td>
<td>4%</td>
</tr>
<tr>
<td>0.50</td>
<td>0.33</td>
<td>2.00</td>
<td>1.67</td>
<td>14%</td>
<td>6.66</td>
<td>-20%</td>
<td>-11%</td>
</tr>
<tr>
<td>0.33</td>
<td>0.26</td>
<td>1.33</td>
<td>1.30</td>
<td>11%</td>
<td>7.40</td>
<td>-23%</td>
<td>-16%</td>
</tr>
<tr>
<td>0.25</td>
<td>0.22</td>
<td>1.00</td>
<td>1.11</td>
<td>10%</td>
<td>7.77</td>
<td>-24%</td>
<td>-19%</td>
</tr>
<tr>
<td>0.10</td>
<td>0.16</td>
<td>0.40</td>
<td>0.78</td>
<td>7%</td>
<td>8.44</td>
<td>-27%</td>
<td>-23%</td>
</tr>
<tr>
<td>0</td>
<td>0.11</td>
<td>0.00</td>
<td>0.56</td>
<td>5%</td>
<td>8.88</td>
<td>-29%</td>
<td>-26%</td>
</tr>
</tbody>
</table>

Note: Positive error percentages indicate underprediction, while negative error percentages indicate overprediction.
Predicting Travel Time During Regime CD

Figure 8 illustrates transition CD from congested state C to uncongested state D (see also Figures 2 and 3). Vehicles to the left are traveling at $v_c$, and a backward-moving recovery wave passes through the section at speed $v_{CD}$. The sensor receives the “uncongested” signal at time $t_r$, which occurs a lag time $\alpha'$ after the wave enters the section. For the sake of brevity, the intermediate computation steps for regime are not shown here, since the sequence is identical to that followed for regime AC, only with different parameters. As shown in the figure, travel time is overpredicted for vehicles entering the section after vehicle $j_1$ and before time $t_r$, and the VHT for these vehicles can be calculated as:

$$ VHT_{\text{pred}}^o = \frac{q_cl}{v_c} \left( \frac{l}{2v_{CD}} - \frac{s}{v_{CD}} \right) $$

$$ VHT_{\text{act}}^o = \frac{q_c}{2} \left( \frac{l}{v_c} - \frac{s}{2v_{CD}} \right) \left( \frac{1}{v_c} + \frac{1}{v_{CD}} \frac{(v_f - v_{CD}) + \frac{1}{2} s (v_c - v_f)}{(v_c - v_{CD})} \right) $$

Vehicles entering after vehicle $j_2$ expect free flow travel times but experience higher travel times (underprediction). Their predicted and actual VHT are:

$$ VHT_{\text{pred}}^u = \frac{q_c}{2v_{CD}v_f} \left( \frac{s}{2} - l \right) $$

$$ VHT_{\text{act}}^u = \frac{q_c}{2v_{CD}v_f} \left( \frac{s}{2} - l \right) \left( \frac{1}{v_c} + \frac{1}{v_{CD}} \frac{(v_f - v_{CD}) + \frac{1}{2} s (v_c - v_f)}{(v_c - v_{CD})} \right) $$

Table 1 shows the under- and overprediction results for regime CD. In this case, the percent VHT error for underprediction increases with increased detection. Figure 9 shows that when the errors are simply added (allowing overprediction to cancel out underprediction) an optimal $s$ would be about 0.4 mi. Applying a 3× penalty to underprediction results in an optimal $s$ of about 0.8 mi. Adding the absolute values of the under- and overprediction results in errors in the 17-19% range.

FIGURE 8 Travel time estimation during regime CD.

Vehicles entering after vehicle $j_2$ expect free flow travel times but experience higher travel times (underprediction). Their predicted and actual VHT are:

$$ VHT_{\text{pred}}^u = \frac{q_c}{2v_{CD}v_f} \left( \frac{s}{2} - l \right) $$

$$ VHT_{\text{act}}^u = \frac{q_c}{2v_{CD}v_f} \left( \frac{s}{2} - l \right) \left( \frac{1}{v_c} + \frac{1}{v_{CD}} \frac{(v_f - v_{CD}) + \frac{1}{2} s (v_c - v_f)}{(v_c - v_{CD})} \right) $$

Table 1 shows the under- and overprediction results for regime CD. In this case, the percent VHT error for underprediction increases with increased detection. Figure 9 shows that when the errors are simply added (allowing overprediction to cancel out underprediction) an optimal $s$ would be about 0.4 mi. Applying a 3× penalty to underprediction results in an optimal $s$ of about 0.8 mi. Adding the absolute values of the under- and overprediction results in errors in the 17-19% range.
Figure 3 shows a freeway section between mileposts 294 and 295, where it is known that freeway travel time estimation can be performed accurately using the midpoint method throughout the entire day except during regimes AC and CD. It is possible to consider just the impact of VHT under-prediction during the transition regimes. Figure 10 shows the additive effects of only underpredicted VMT (data from regimes AC and CD in Table 1). The optimal s when considering only underprediction is 0.5 mi (minimum is 16.5% error).

Table 1 also shows the overall additive effects of the underprediction and overprediction that occurs in regimes AC and CD. As shown for $\lambda = s = 1$, the aggregate effect is a 2% VHT overprediction error. This error increases to 12% VHT overprediction for $s = 0.10$. This is also shown graphically in Figure 11. Since drivers may assign a higher value to over-prediction, a 3× penalty is applied to the underprediction error before it is added to the over-prediction error, resulting in a net 14% underprediction for $s = 1$ mi, and a 3% overprediction for $s = 0.1$ mi. As shown in Figure 11, this reveals an optimal $s$ of 0.33 mi (zero error). It is not clear how differently drivers actually value underprediction versus overprediction, so this is just a sample, and could be the topic of further research. The total error applying the sum of the absolute value of the underprediction and overprediction error is also shown in Table 1 (18-22% error) and in Figure 11.
CHANGING DETECTOR LOCATIONS

The results presented so far for a single detector are dependent, of course, on the assumption that the detector is located in the middle of the link (hence the “midpoint” method). We investigate here what would be different if the detector were located at the far downstream end of the link. Of course, this change would not be effected by actually moving the detector, but rather redefining the logical endpoints of the link. Rather than repeat the analysis of the entire paper, we will repeat here just that part corresponding to the AC transition. It should be clear to the interested reader at that point how to repeat the analysis for other shock regimes, as well as for other detector locations not previously considered.

FIGURE 12 Transition example during regime AC with detector at downstream end.

Figure 12 shows the AC shock regime situation when a single detector is located on the downstream end of the link. This is in contrast with the situation shown in Figure 5. Here we will repeat the analysis that started with equation (1), for travel time underestimation, followed by overestimation. We still focus on the vehicle labeled \( j_2 \), which is the last vehicle to enter the section without the benefit of information from the sensor. In this case, the vehicle travels a distance

\[
\frac{v_f l}{v_f - v_{AC}}
\]
at speed \( v_f \) and the remainder of the section at speed \( v_c \). The travel time \( z \) for vehicle \( j \) is therefore:

\[
z = \frac{v_f l}{v_f - v_{AC}} + \frac{l - v_f l}{v_f - v_{AC}} = \frac{l}{v_f - v_{AC}} + \frac{v_f l - v_{AC} l - v_f l}{v_f - v_{AC}}
\]

\[
= \frac{v_f l}{v_c (v_f - v_{AC})} - \frac{v_{AC} l}{v_c (v_f - v_{AC})} + \frac{l (v_c - v_{AC})}{v_c (v_f - v_{AC})} - \frac{l}{v_f - v_{AC}}
\]

(14)

Because the sensor is at the upstream end, there is no lag \((\alpha = 0)\) and the free-flow travel time is as before. The maximum prediction error \( u_{\text{max}} \) is therefore:

\[
u_{\text{max}} = z - l = \frac{l (v_c - v_{AC})}{v_c (v_f - v_{AC})} - \frac{l}{v_f}
\]

(15)

The time span over which underpredicted vehicles enter the link is the free-flow travel time plus the lag, which is zero. Again, we count vehicles using the upstream boundary as a reference, and at this location the flow is \( q_A \); hence the number of vehicles in this condition is:

\[
q_A \left( \frac{l}{v_f} \right)
\]

(16)

Each of these vehicles has predicted travel time equal to the free-flow travel time, so the total VHT per hour predicted for those vehicles who ultimately end up with underpredicted travel times is:

\[
VHT_{\text{pred}}^{u} = q_A \left( \frac{l}{v_f} \right) \left( \frac{l}{v_f} \right) = q_A \frac{l^2}{v_f^2}
\]

(17)

The actual travel times range from the free-flow travel time to the worst-case scenario given in (14); hence the actual VHT per hour is given by:

\[
VHT_{\text{act}}^{u} = q_A \left( \frac{l}{v_f} \right) + \frac{l (v_c - v_{AC})}{v_f (v_f - v_{AC})}
\]

(18)

For vehicles in this situation whose travel time is over-predicted, the time period over which they cross the upstream end of the link is:

\[
- \frac{l}{v_{AC}}
\]

Again, the flow at this location is \( q_A \), so the number of vehicles in this condition is:

\[
- \frac{q_A l}{v_{AC}}
\]

(19)

Each of these vehicles expects to travel the link at speed \( v_c \), so the total predicted VHT per hour is:

\[
VHT_{\text{pred}}^{o} = \frac{q_A l^2}{v_{AC} v_c}
\]

(20)

The actual VHT per hour is:

\[
VHT_{\text{act}}^{o} = \frac{q_A l}{2v_{AC}} \left( \frac{l}{v_c} + \frac{l (v_c - v_{AC})}{v_c (v_f - v_{AC})} \right)
\]

(21)

Table 2 shows how these new results compare with those from the midpoint method, for a single detector and for regime AC only. Not surprisingly, situating the detector on the downstream end allows the shock information to be known much earlier, therefore, there is a lower tendency to under-predict travel times but a greater tendency to over-predict. Other detector locations could also be tested for specific situations, using the same process shown here but with different details pertaining to detector location.
CONCLUSIONS

This paper addressed the question: “how much detection do you need?” in the context of accurate estimation of freeway travel time using the midpoint method. It is false to assume that detection decisions are made in isolation of other issues, and in fact, sensors are usually placed to enable operation of ramp metering (e.g. Portland) and traffic monitoring (e.g., counting, speed maps, and incident detection). Freeway travel time estimation is often a useful side benefit that can be leveraged from an existing sensor network. It is also false to assume that in the future some combination of fixed infrastructure based sensors and vehicle based sensing (e.g. AVL) will not provide answers and improvements. However, this analysis has taken the question of sensor density in some degree of isolation which has resulted in some helpful outcomes. Another issue that has been left for further research is the question of where to optimally place sensors, beyond simply a question of spacing. The use of other travel time algorithms beyond the midpoint method should also be explored further. The optimal placement of sensors in relation to known bottlenecks, and high incident locations will be examined in the future.

Even were these results to be applied directly, the method does not culminate in a single “optimal” answer because tradeoffs exist in the space blending drivers’ experiences and expectations. Research might be undertaken to reveal some preference structure and illuminate unknown parameters, and it might also be necessary to apply some level of professional judgment and/or policy imposition to the decision. The proper balance between under- and over-prediction, for example, is not known, and may not be the same for all drivers. Ultimately, the goal of this paper has been to highlight these issues and to put the quantitative aspects of the problem on sound footing.

ACKNOWLEDGEMENTS

Galen McGill of ODOT posed the sensor density question and supports our freeway travel time research. Dennis Mitchell and Jack Marchant of ODOT provide the data. K. Tufte, S. Kothuri, B. Zielke and R. Fernandez of PSU assisted in development of this work.

REFERENCES

