Impacts of Sensor Spacing on Accurate Freeway Travel Time Estimation for Traveler Information

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ABSTRACT

Travel time estimation is a critical ingredient for transportation management and traveler information—both infrastructure-based and in-vehicle. Infrastructure managers and operators are interested in estimating optimal freeway sensor density for new construction and retrofits. Focusing on freeway travel time estimation for display on roadside variable message signs, this paper describes a concept developed from first principles of traffic flow for establishing optimal sensor density. The method is based on computing the magnitude of under- and overprediction of travel time during shock passages when using the midpoint method. Two case studies are presented considering representative traffic dynamics situations. Along with other performance metrics, a suggested aggregate measure developed from vehicle hours traveled (VHT) is described for a reasonable range of detector densities. Extensions of the method to account for both recurrent and nonrecurrent congestion are included. Finally some suggestions for future research are described.

Keywords: traveler information, traffic management, freeway sensor

INTRODUCTION

Travel time estimation is used for a wide range of applications, including planning, design, performance measurement, operational analysis, traffic management, incident detection, navigation, traveler information, and more. In the U.S., freeways account for 3% of the national highway mileage, but accommodate more than 30% of the vehicle-miles traveled (VMT). The alleviation of congestion on urban freeways continues to receive substantial attention, and transportation agencies are implementing improved management strategies to reduce congestion and improve travel time reliability. Accurate freeway travel time measurement is important for improving traffic management and for enabling better informed traveler decisions. For example, incident management and traveler information systems can be implemented at relatively low cost. However, such systems rely upon accurate measurement of traffic parameters such as flow, speed, travel time, delay and reliability. Usually these data are measured by fixed sensors (loop detectors, video cameras, microwave detectors, radar sensors, etc.) or by mobile data sources such as automatic vehicle identification (AVI) toll tags or automatic vehicle location (AVL) probe vehicles. In the future, technologies such as vehicular ad hoc networks will likely provide a supplemental data source. It is possible that some applications using travel time are not affected by error; however this paper focuses on a specific application where travel time error is an issue.

Collecting relevant data as input to travel time computations is one ingredient for the successful transmission and display of travel time information for travelers. This can be done via a speed map showing color coded freeway segments or origin-destination listings available via the Internet, handheld devices such as PDAs and mobile phones, or via roadside variable message signs (VMS). While there are many potential uses and display mechanisms for travel time information, the specific focus of this paper is the situation where a driver receives travel time information based on data from fixed sensors upon entering a freeway section via a VMS.

As motivation for this paper, Figure 1 shows a time-space speed plot from an actual freeway corridor (Northbound I-5 in Portland, Oregon on October 31, 2006), with the x-axis as time, the y-axis as distance (in miles), and the “color” indicating speed. The Oregon Department of
Transportation (ODOT) has installed loop detectors at an average spacing of 1.2 miles and five VMS along this corridor (locations are shown on the right y-axis of the figure). ODOT displays travel time information to Downtown on VMS 1, 2 and 3 during the AM peak period. ODOT and others are interested in increasing sensor density toward some “optimal” value, in order to improve the accuracy of travel time information in a cost-effective manner. In order to answer the question “how much detection is needed,” this paper focuses on the common use of fixed sensors (such as loop detectors) as a basis for formulating an optimal detector spacing strategy.
A hypothetical 1-mile segment such as the one between MP 294 and 295 in Figure 1 will be considered as a basis for the analysis. Sensor placement has been the topic of several recent studies (see, for example, Bartin, et al., 2006; Fujito, et al., 2006; Leow et al., 2008).

Turning away from the specific example shown in Figure 1, and in order to simplify and generalize the situation using a consistent analytical framework, Figure 2 illustrates a hypothetical time-space (t-x) plane of time interval \( t_1 \) and length \( \ell \) with a VMS located at the upstream end. Hypothetical vehicle trajectories are shown (in grey) with that of vehicle \( i \) highlighted in black. If equipped with AVL or any other data logging system, vehicle \( i \)'s trajectory could be constructed and all information necessary to completely describe its path would be known, including its speed (slope of trajectory) at any point and its actual travel time. With an assumed free flow speed, the free flow travel time is known and the delay (actual minus free flow travel time) for vehicle \( i \) can be computed. In practice it is more common to use sensors at fixed points (such as point \( x_1 \)) to measure speed and subsequently extrapolate that speed over a segment. In Figure 2 a speed measured at \( x_1 \) is extrapolated over a segment of length \( \ell \) resulting in the calculation of an extrapolated travel time. The estimate does not perfectly match the actual travel time (in this case it is overpredicted). The magnitude of this difference, as a function of detector density, is the topic of interest in this paper in the context of providing travel time information to vehicles entering the section via a VMS. The important topic of sensor errors is not considered here, nor are other issues related to the calculation or communication of travel times for other purposes. The passage of a shock represents the “worst case scenario” for travel time prediction, so the methods presented in this paper can be interpreted as a form of robust decision analysis about sensor spacing.

**TRAVEL TIME ESTIMATION FRAMEWORK**

A simplified framework is assumed as a platform for this analysis. In freeway dynamics there are a variety of transition types as shown in Figure 3A (see May, 1990), some of which move forward and others that move backward. The backward forming shock affects travel time calculations at the onset of congestion since the shock moves against traffic. A forward forming wave travels with traffic and may have less effect. For this analysis, a fundamental traffic flow relation has been assumed, as shown in Figure 3B. In particular, a triangular shape is used; this is common in the literature when broad questions need to be addressed with a minimum of complications. A congested state \( C \) (flow \( q_c \) and speed \( v_c \)) and uncongested states \( A, B, D, \) and \( E \) (flows \( q_A, q_B, q_D, q_E \), speed \( v_f \)) are shown. In order to include the maximum number of representative traffic dynamics situations, two cases are analyzed, shown in Figures 3C and 3D. For Case 1, Figure 3C shows a t-x plane showing a bottleneck (either recurrent or nonrecurrent) at location \( bn \). It is assumed, for the sake of simplicity, that the bottleneck state is binary; it is either active, with some reduced capacity captured by state \( C \), or it is inactive, in which case the nominal capacity can be permitted. Following the rules of standard first-order macroscopic traffic dynamics, and assuming the nominal traffic state was \( A \), there is a transition between uncongested state \( A \) and congested state \( C \), marked by a shock of velocity \( v_{AC} \). Figure 3C shows that for an assumed arbitrary highway segment (separated by the two dashed lines with a VMS at the upstream end) transition \( AC \) is bounded by a rectangle as the shock passes. If this were an active bottleneck that was deactivated at time \( t_{deact} \), then transition \( CD \) would occur, between
congested state C and uncongested state D, marked by a backward-moving recovery wave of velocity \( v_{CD} \). Transition DA (the return to nominal conditions) is marked by a forward-moving recovery wave of velocity \( v_f \). There is no requirement that these three transitions always occur in this configuration or order, and the computations later in the paper assess each type of state transition separately. To expand the analysis, Figure 3D shows Case 2, which includes a backward forming transition (AC) similar to Case 1 and a forward recovery (CE) marked by a forward-moving recovery wave of velocity \( v_{CE} \). Later computations will examine travel time estimation during transitions AC, CD, DA and CE.

The first-order hydrodynamic model approximates actual vehicle trajectories, such as those in Figure 2, with piecewise-linear trajectories. For the purposes of travel time measurements over an assumed freeway link, this is irrelevant, because one vehicle’s travel time is measured as the horizontal difference between the endpoints of its trajectory on the link, and the microscopic details of the shapes of speed transitions in the vicinity of a shock do not matter in this analysis.

**CALCULATING TRAVEL TIME DURING STATIONARY CONDITIONS**

As noted above, travel time information is used for purposes other than VMS display, and for many of these purposes the error structure is quite different. In particular, for measuring VHT for system evaluation, delays are summed over a rectangular time-space domain and any lag time does not lead to error. For these applications, the errors are a strictly increasing function of the detector spacing, so optimal detector spacing is not an issue. However, the purpose of this paper is to consider travel time information that would be provided to travelers using VMS.

There are situations when travel time can be estimated perfectly well. Referring to Figure 1, travel times during regime 1 (fully uncongested) and regime 2 (fully congested) can be estimated
without error. Viewing the entire $t-x$ plane in Figure 1, it is clear that travel times can be correctly estimated during fully uncongested or congested periods (regimes 1 and 2), but there are notable transitions in and out of congested states—which are of interest for this analysis.

From Figure 2, travel times in states A, B, D and E (uncongested), C (congested) and during transition DA (uncongested) can be estimated without error over an arbitrary section of length $\ell$, by:

$$tt_f = \ell / v_f, \quad tt_c = \ell / v_c$$

where $tt_f$ is free flow travel time and $tt_c$ is congested travel time. If the sensor where speed is measured is located at the center of the segment, this would be called the midpoint method. Travel time estimation over a segment can be performed accurately if the traffic state within the segment is either fully uncongested or fully congested. In addition, $tt_c$ is an upper bound on the actual travel time through the segment and $tt_f$ is a lower bound on the actual travel time. Empirical studies have shown that travel time estimation during stationary conditions (either uncongested or congested) is usually quite accurate (Cortes, et al., 2002; Monsere, et al., 2006). For simplicity, in this paper, the midpoint method will be used; however, other methods have been developed for improving travel time estimates (Coifman, 2002; Wang and Liu, 2005; Kwon, et al., 2007; Liu and Danczyk, 2008; Ni and Wang, 2008) and can be studied further later.

**TRAVEL TIME ESTIMATION DURING TRANSITIONS**

As initially illustrated in Figure 3A, there are two basic transition types in freeway traffic flow: uncongested to congested (e.g. AC) and the reverse (e.g. CD or CE). Transitions can move forward or backward and can occur multiple times in a given section as queues propagate and dissipate and sometimes combine with one another. Queues may be caused by recurrent bottlenecks or by nonrecurrent conditions such as incidents. These transient conditions induce errors in travel time estimations based on current measured conditions, and depending on the specifics, can result in either overprediction or underprediction. In the specific travel time information context considered here, underprediction is considered to be more problematic than overprediction, since travelers whose travel times are much longer than predicted at their entry to a segment will be more likely to be dissatisfied. The method involves choosing a sensor spacing as a result of a tradeoff between over- and underprediction and for the reasons cited above, underprediction will be given a higher weight in this tradeoff.

This paper provides an analytical framework and also includes a series of numerical examples using a hypothetical segment of length $\ell = 1$ mile for comparison purposes. For the numerical examples a range of five sensor spacings $s$ (0.1 to 1 mile) will be used and assumed traffic flow parameters will be: $q_A = 2000$ vphpl, $q_C = 1800$ vphpl, $q_E = 1600$ vphpl, $v_f = 60$ mph, $v_c = 30$ mph, $v_{CD} = -17.1$ mph, $v_{AC} = -7.5$ mph, and $v_{CE} = +6.0$ mph. For the calculations that follow, we will consider as the domain of the travel time computations a region of the time-space plane that is bounded by the link endpoints, and includes exactly those vehicles whose trajectories were affected by the passage of the shock. It is also assumed that travel time information can be transmitted to vehicles entering the hypothetical segment via a VMS located at the upstream end of the segment (this concept could be broadened to an in-vehicle or handheld device). Several performance measures are used when comparing results for the range of sensor spacings considered, which will be described below.
Underpredicting Travel Time During Transition AC

Figure 4 illustrates transition AC from uncongested conditions with vehicles traveling at $v_f$, to a congested state with vehicles traveling at $v_c$. The figure illustrates the backward moving shock passing through the segment of length $\ell$ with an initial sensor spacing $s$, at a speed $v_{AC}$. Trajectory $j_1$ is the last trajectory to pass through the section at speed $v_f$ and trajectory $j_3$ is the first to traverse the section at speed $v_c$. All vehicles between $j_1$ and $j_3$ change speeds at some point in the section, and their average speeds are therefore between $v_f$ and $v_c$. The shading in the figure encapsulates these vehicles. The figure also shows that the sensor continues to record speed $v_f$ until time $t_c$, a lag time $\alpha = s / 2v_{AC}$ after the shock actually enters the section. After vehicle $j_1$ and until $t_c$, based on the VMS message, drivers would expect a free flow trip time through the entire section while their actual trip time will be higher. For example, driver $j_2$ enters the section an instant before time $t_c$. Based on the VMS, $j_2$ expects a speed of $v_f$ through the entire section, but experiences a longer actual travel time $z$. For vehicles entering the section after $j_1$ but before time $t_c$, travel time is underpredicted by an amount equal to the difference between the expected travel time (dashed trajectory in the figure) and the actual travel time (solid trajectory). Prior to encountering the shock, vehicle $j_2$ travels a distance:

$$x_{AC} = \frac{v_f \left( \ell - \frac{1}{2}s \right)}{v_f - v_{AC}}$$

(1)

at speed $v_f$, and drives the remainder of the link at speed $v_c$. Thus, the travel time experienced by vehicle $j_2$ is:

$$z = \frac{v_f \left( \ell - \frac{1}{2}s \right)}{v_f - v_{AC}} + \frac{\ell - v_f \left( \ell - \frac{1}{2}s \right)}{v_f - v_{AC}}\frac{v_f}{v_c} + \frac{\ell - \frac{1}{2}s}{v_f - v_{AC}}\frac{v_f \left( v_f - v_{AC} \right) - v_f \left( \ell - \frac{1}{2}s \right)}{v_c \left( v_f - v_{AC} \right)}$$

(2)
The amount by which travel time is underpredicted is a maximum for this vehicle, and this maximum error is given by:

$$u_{\text{max}} = z - \tau_f = \frac{\ell(v_f - v_{AC}) + \sqrt{s}(v_f - v_e)}{v_f - v_{AC}} - \frac{\ell}{v_f}$$  \hspace{1cm} (3)$$

For example, if $\ell = s = 1$, from the VMS the driver of vehicle $j_2$ expects a travel time of 1 minute, but actually experiences a 1.56 minute trip, more than 50% longer than expected. If one assigns zero weight to any travel time overprediction (between time $t_c$ and the time the shock reaches the upstream end of the section), for now one can neglect any flow entering the section after time $t_c$. The remainder of this section only considers vehicles for which travel time is underestimated, since it is assumed that traffic management officials want to avoid giving drivers false expectations of low travel times, when they actually experience longer ones. In addition to considering as performance measures the values of maximum underprediction ($u_{\text{max}}$), lag time ($\alpha$), and percent error, one can quantify the predicted and actual vehicle-hours traveled ($VHT_{\text{pred}}$ and $VHT_{\text{act}}$) solely for vehicles experiencing underprediction (superscript $u$) over the hypothetical segment. Recall that the errors reported would be only for the transitions, since when traffic conditions are homogeneous it is assumed that travel times can be computed without error.

There is an assumption here (as throughout other forms of travel time analysis) that individual vehicles’ travel times are additive. In reality, different travelers have different values of time, but this is a common practice in transportation analyses. To compute the $VHT_{\text{pred}}$ and $VHT_{\text{act}}$ for comparison, the number of vehicles is counted in each situation, and is multiplied by the average travel time (either predicted or actual). Since the trajectories and shocks are all assumed to be linear, the average travel time for a group of vehicles with a similar disposition will always be the midpoint of the best and worst travel times amongst that set.

During transition AC, the trajectories for which travel time is underpredicted cross the upstream end of the link over a time span of:

$$t_{AC} = \tau_f + \alpha = \frac{\ell}{v_f} - \frac{s}{2v_{AC}}$$  \hspace{1cm} (4)$$

At this location, the flow is $q_A$; hence the number of vehicles in this condition is:

$$n_{AC} = q_A \left( \frac{\ell}{v_f} - \frac{s}{2v_{AC}} \right)$$  \hspace{1cm} (5)$$

Based on the VMS, each of these vehicles expected to travel at free flow speed and therefore have free flow travel time across the link, so the predicted VHT for trajectories for which travel time was underestimated in the AC transition is:

$$VHT^u_{\text{pred}} = q_A \frac{\ell}{v_f} \left( \frac{\ell}{v_f} - \frac{s}{2v_{AC}} \right)$$  \hspace{1cm} (6)$$

The actual total travel time can be found by multiplying the same number of vehicles by their expected travel time, which is midway between the highest and lowest travel times for this group of vehicles. The lowest travel time is the free-flow travel time, $\tau_f = \ell/v_f$. The vehicle with the
highest travel time is trajectory $j_2$ of Figure 4, whose travel time was computed in equation (2). The average travel time for all vehicles is then the average between these lowest and highest values, and the total actual travel time for these vehicles, whose travel time was underpredicted, can be found by multiplying this average by the number of vehicles in equation (5), resulting in the following:

$$VHT_{act}^w = \frac{Q_d}{\ell} \left( \frac{\ell}{v_f} - \frac{s}{2v_{AC}} \right) \left( \frac{\ell}{v_f} + \frac{\ell(v_e - v_{AC}) + \sqrt{2s(v_f - v_e)}}{v_u(v_f - v_{AC})} \right)$$  \hspace{1cm} (7)

For $\ell = s = 1$, the predicted VHT/mile is 2.78 veh-hr/mile, yet the actual VHT/mile is 3.55 veh-hr/mile, a 22% error over the collection of vehicles entering transition AC before $t_c$. Vehicles experience actual underpredictions between 0 and 0.56 minutes.

In order to extend these calculations to arrangements with greater sensor density, Figure 5 illustrates how predicted and actual VHT will change with additional sensors. The increased sensor placement reduces the lag time $\alpha$ so vehicles entering the segment receive the congestion message sooner via the VMS. The speed from each detector will be applied only to a portion of $\ell$ rather than to the whole section, which reduces the magnitude of the VHT composed of travel time underprediction. The figure indicates this using the darker shaded areas. As the lag time decreases, the VHT of traffic impacted by travel time underprediction will decrease in this section. For now the issue of overprediction is set aside.

Table 1 shows the values of $u_{max}$ and lag time $\alpha$ for transition AC as a function of detector spacing $s$. The table also shows the predicted and actual VHT/mile for a range of $s$. To reiterate, for this hypothetical 1 mile segment, vehicles entering the section prior to $t_c$ will expect free flow conditions based on the VMS, but will instead experience progressively longer travel times (the lag time gets shorter with increased detector density). For this set of vehicles, the actual VHT is higher than the predicted VHT. Even with sensors at 0.1 mile spacing, the VHT error is 7% and vehicles expecting a travel time of 1 minute actually experience 1.16 minutes, a 16% underprediction. The gap between the predicted and actual VHT grows with larger sensor spacing. For the range of sensor spacing considered, the VHT error falls between 7% and 22%.

![Figure 5](image-url) Travel time estimation during regime AC.
For the average ODOT sensor spacing of 1.2 mile, the VHT error would be 24%. In this case, more detection is better if the goal is to minimize underprediction. Spacing between 0.25 and 0.50 mile would keep \( u_{\text{max}} \) below 33% of \( t_f \). Table 1 also includes a sixth row with \( s = 0 \), which is equivalent to a form of ubiquitous sensor coverage.

**Overpredicting Travel Time During Transition AC**

The previous section examined the impact of only the travel time underprediction during transition AC. Travel times for vehicles entering the section after time \( t_c \) and until the shock crosses the upstream end of the link are overpredicted. The duration over which these vehicles cross the upstream end of the link is:

\[
t_{AC} = \left( \frac{1}{v_{AC}} \right) \left( \frac{\sqrt{2} s - \ell}{2} \right)
\]

and they do so at flow \( q_A \). The number of such vehicles is therefore:

\[
n_{AC} = \frac{q_A}{v_{AC}} \left( \sqrt{2} s - \ell \right)
\]

and based on the VMS, each expected the congested travel time \( \ell/v_C \). For transition AC, including only the overprediction, the predicted VHT (with superscript \( o \)) is:

\[
VHT_{\text{pred}}^{o} = \frac{q_A}{v_{AC} v_C} \left( \sqrt{2} s - \ell \right)
\]

The actual VHT for these vehicles can be computed in a manner identical to equation (7), except that the number of vehicles in this condition is given by equation (9) and the two extreme travel times whose average is the average for all vehicles are \( \ell/v_C \) and the travel time for vehicle \( j_2 \) as shown in equation (3). This results in the following total travel time for vehicles in transition AC, whose travel time was overpredicted:

\[
VHT_{\text{act}}^{o} = \frac{q_A}{2 v_{AC}} \left( \sqrt{2} s - \ell \right) \left( \frac{\ell}{v_C} + \frac{(v_{AC} - v_c) + \sqrt{2} s (v_j - v_c)}{v_C (v_j - v_{AC})} \right)
\]

Table 1 shows that for \( \ell = s = 1 \), for vehicles entering the section after \( t_c \) the predicted VHT is 4.44 veh-hr/mile, and the actual VHT is 3.95 veh-hr/mile, an overprediction of 13%. Many drivers would be pleasantly surprised by the shorter travel time but it is not safe to assume that overprediction is always benign. In a dynamic route choice application, for example, it could disguise a better link and thus lead to suboptimal route recommendations. Travelers could lose their trust in the system. With increased sensor density, the percent error in VHT increases to a maximum of 27% overprediction for \( s = 0.10 \).

When viewing Figure 6 and Table 1 together, for transition AC it can be seen that when \( s \approx 0.5 \), the underprediction error is 14%, the overprediction error is 20%, the aggregate error is −11% (overprediction), and the absolute error is 18%. These errors are only for transition AC, recalling that during homogeneous traffic states, travel time estimates can be made without error. This might represent a reasonable compromise for users’ acceptance.
As an extension of the above, if one assumes that under- and overpredicted VHT can be added together numerically (with one canceling out the other), the total predicted VHT in transition AC would be $VHT^{u}_{\text{pred}} + VHT^{o}_{\text{pred}}$, and the actual VHT in transition AC would be...
\( VHT^{o}_{act} + VHT^{o}_{act} \) (7.50 veh-hr/mile). Here one time unit of underprediction is canceled out by a time unit overprediction. As shown in Table 1, when the two components are added, for \( \ell = s = 1 \), the effect is still a 4\% underprediction in VHT. For decreasing values of \( s \), the aggregate effect is overprediction, up to 23\% with \( s = 0.10 \). Figure 6 illustrates this graphically; here a spacing of \( s \approx 0.5 \) mile is near where the VHT error line crosses from overall underprediction to overprediction. For the situation where drivers might assign a higher “price” to underprediction than to overprediction, Figure 6 shows, with the line series labeled “penalty,” the percent error in VHT when the underpredicted VHT is weighted at 3\times that of overpredicted VHT. From this example, a value of \( s \approx 0.5 \) mile would be reasonable. Table 1 also includes a column where the total VHT error is considered by adding the absolute value of the differences in actual and predicted VHT (also shown in Figure 6). The absolute error ranges between 17\% and 25\% for the range of detector spacing considered. Other approaches to the 3\times penalty could be explored in the future.

**Predicting Travel Time During Transition CD**

Case 1 in Figure 3C also includes transition CD from congested state C to uncongested state D, shown in Figure 7. Vehicles from the left are traveling at \( v_c \), and a backward-moving recovery wave passes through the section at speed \( v_{CD} \). The sensor receives the “uncongested” signal at time \( t_r \), which occurs a lag time \( \alpha' \) after the wave enters the section. For the sake of brevity, the intermediate computational steps for transition CD are not shown here, since the sequence is identical to that followed for transition AC, only with different parameters. As shown in the figure, travel time is overpredicted for vehicles entering the section after trajectory \( j_1 \) and before time \( t_r \). The VHT for these vehicles can be calculated as:

\[
VHT^o_{pred} = \frac{q_c \ell (\ell - s)}{v_c (v_c - 2v_{CD})} \quad (12)
\]

\[
VHT^o_{act} = \frac{q_c}{2} \left( \frac{\ell}{v_c} - \frac{s}{2v_{CD}} \right) \left( \frac{\ell}{v_c} + \frac{\ell (v_f - v_{CD}) + \frac{1}{2} s (v_c - v_f)}{v_f (v_c - v_{CD})} \right) \quad (13)
\]

![Figure 7 Travel time estimation during regime CD.](image-url)
Vehicles entering after vehicle $j_2$ expect free flow travel times but experience higher travel times (underprediction). Their predicted and actual VHT are:

$$VHT_{\text{pred}}^u = \frac{q_c}{v_{CD}} \left( \frac{q_f}{v_f} \left( \frac{\ell}{2} - \ell \right) \right)$$

$$VHT_{\text{act}}^u = \frac{q_c}{2v_{CD}v_f} \left( \frac{q_f}{v_f} \left( \frac{\ell}{2} - \ell \right) \left( \frac{f(v_f - v_{CD}) + s(v_c - v_f)}{v_c - v_{CD}} \right) \right)$$

Table 1 shows the under- and overprediction results for transition CD. In this case, the percent VHT error for underprediction increases with increased detection. Figure 8 shows that when the errors are simply added (allowing overprediction to cancel out underprediction) an optimal $s$ would be between 0.33–0.5 mile. Applying a $3 \times$ penalty to underprediction results in an optimal $s$ between 0.5–1.0 mile. Adding the absolute values of the under- and overprediction results in errors in the 17–19% range.

![Figure 8](image-url)  
Figure 8  Travel time over- and underprediction in regime CD (VHT/mile).

**Combined Effects of Transition Transitions AC and CD**

For Case 1, Figure 3C shows a hypothetical freeway section where it is assumed that freeway travel time estimation can be performed accurately using the midpoint method throughout the entire day except during transitions AC and CD. It is possible to consider just the impact of VHT underprediction during the transitions. Figure 9 shows the additive effects of only underpredicted VMT (data from transitions AC and CD in Table 1). The optimal $s$ when considering only underprediction is 0.5 mile (minimum is 16.5% error during the transitions).

Table 1 also shows the overall additive effects of the underprediction and overprediction that occurs in transitions AC and CD. As shown for $\ell = s = 1$, the aggregate effect is a 2% VHT overprediction error. This error increases to 12% VHT overprediction for $s = 0.10$. This is also shown graphically in Figure 10. As before, a $3 \times$ penalty is applied to the underprediction error
before it is added to the overprediction error, resulting in a net 14% underprediction for $s = 1$ mile, and a 3% overprediction for $s = 0.1$ mile.

As shown in Figure 10, this reveals an possibly optimal $s$ of 0.33 mile (zero error). It is not clear how differently drivers actually value underprediction versus overprediction, so this is merely a sample, and could be the topic of further research. The total error applying the sum of the absolute values of the underprediction and overprediction errors is also shown in Table 1 (18–22% error) and in Figure 10.
**PREDICTING TRAVEL TIME DURING TRANSITION CE**

Case 2 from Figure 3D includes transitions AC and CE. Using the logic for transitions AC and CD, with Figure 3D as a reference, Figure 11 shows transition CE from congested speed $v_c$, to uncongested speed $v_f$. Transition CE is a forward moving recovery wave passing through a segment of length $\ell = 1$ mile, sensor spacing $s$, at speed $v_{CE}$. Trajectory $j_1$ represents the last vehicle that traveled at speed $v_c$ through the section, and trajectory $j_3$ represents the first vehicle to traverse the section at speed $v_f$. Trajectories between $j_1$ and $j_3$ (shading in the figure) reduce their speeds in the section, and the average speed is between $v_c$ and $v_f$. The sensor measures speed $v_c$ until time $t_r$, a lag time $\alpha = s/2v_{CE}$ after the wave enters the section. After trajectory $j_1$ and until $t_r$, a congested trip time is predicted through the section while actual trip times will be lower. For example, trajectory $j_2$ enters the section just before $t_r$ with an expected speed of $v_c$ through the section, but experiences a shorter travel time $z$.

Prior to encountering the wave, trajectory $j_2$ travels a distance:

$$x_{CE} = \frac{sv_f}{2(v_f - v_{CE})}$$

at speed $v_f$ and traverses the remainder of the link at speed $v_c$. Thus, the travel time experienced by trajectory $j_2$ is:

$$z = \frac{\ell(v_f - v_{CE}) + \frac{1}{2}s(v_c - v_f)}{v_c(v_f - v_{CE})}$$

The amount by which travel time is overpredicted is maximal for trajectory $j_2$, given by:

$$o_{\text{max}} = z - tt_f = \frac{\ell(v_f - v_{CE}) + \frac{1}{2}s(v_c - v_f)}{v_c(v_f - v_{CE})} - \frac{\ell}{v_f}$$

If $\ell = s = 1$, trajectory $j_2$ would expect a travel time of 1 minute, but actually experience a 1.44 minute trip, 44% longer than expected based on the VMS. As before, the trajectories for which

![Figure 11 Travel time estimation during regime CE.](image-url)
travel time is overpredicted cross the upstream end of the link over a time span equal to the lag time $\alpha$ shown in Figure 11:

$$t_{CE} = \alpha = \frac{s}{2v_{CE}} \quad (19)$$

At this location, the flow is $q_E$; hence the number of vehicles in this condition is

$$n_{CE} = \frac{q_E s}{2v_{CE}} \quad (20)$$

Each of these vehicles was expected to travel at congested speed so the predicted VHT for vehicles whose travel time was overestimated in transition CE is:

$$VHT_{pred} = \frac{q_E \ell s}{2v_c v_{CE}} \quad (21)$$

The actual total travel time can be found by multiplying the same number of vehicles by their expected travel time:

$$VHT_{act} = \frac{q_E \ell}{2v_c} \left( \frac{s}{2v_{CE}} \left( \frac{\ell}{v_c} + \frac{\ell(v_c - v_{CE})}{v_c} \right) \right) + \frac{1}{2}s(v_c - v_f) \quad (22)$$

For $\ell = s = 1$, the predicted VHT/mile is 4.44 veh-hr/mile, and the actual VHT/mile is 3.83 veh-hr/mile, reflecting a 16% error over the collection of vehicles entering transition CE before $t_r$. Vehicles experience actual underpredictions between 0 and 0.44 minute.

Table 2 shows the values of $u_{max}$, lag time $\alpha$, and predicted and actual VHT/mile for transition CE as a function of $s$. For trajectories before $t_r$, the actual VHT is lower than the predicted VHT. With sensors at 0.1 mile spacing, the VHT error is 1% and vehicles expecting a 2 minute travel time actually experience 1.94 minute, a 3% overprediction. The gap between the predicted and actual VHT grows with larger sensor spacing. For the range of sensor spacing considered, the VHT error falls between 1% and 16%. For the average ODOT sensor spacing of 1.2 miles, the VHT error is 20%. If the goal is to minimize overprediction, more detection is better.

Having examined the impact of travel time overprediction during transition CE, Figure 11 shows that travel time underprediction occurs for trajectories entering the section after time $t_r$ until the wave crosses the downstream end of the link. The duration over which these vehicles cross the upstream end of the link at flow $q_E$ is:

$$t_{CE_2} = \left( \frac{2\ell - s}{2v_{CE}} - \frac{\ell}{v_f} \right) \quad (23)$$

The number of such vehicles is:

$$n_{CE_2} = q_E \left( \frac{2\ell - s}{2v_{CE}} - \frac{\ell}{v_f} \right) \quad (24)$$

each expecting the free flow travel time $\ell/v_f$. For transition CE, including only the underprediction, the predicted VHT (with superscript $u$) is:
Table 2: Segment travel time performance measures during regime CE and combined AC and CE

<table>
<thead>
<tr>
<th>Regime CE</th>
<th>Overprediction (before ( t_r ))</th>
<th>Underprediction (after ( t_r ))</th>
<th>CE Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>s ( (\text{mi}) )</td>
<td>( u_{\text{max}} ) ( (\text{min}) )</td>
<td>Lag Time ( (\text{min}) )</td>
<td>VHT/mile</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------------------------</td>
<td>----------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>1.00</td>
<td>0.44</td>
<td>4.00</td>
<td>4.44</td>
</tr>
<tr>
<td>0.50</td>
<td>0.72</td>
<td>2.50</td>
<td>2.22</td>
</tr>
<tr>
<td>0.33</td>
<td>0.82</td>
<td>1.67</td>
<td>1.48</td>
</tr>
<tr>
<td>0.25</td>
<td>0.86</td>
<td>1.25</td>
<td>1.11</td>
</tr>
<tr>
<td>0.10</td>
<td>0.94</td>
<td>0.50</td>
<td>0.44</td>
</tr>
<tr>
<td>0</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 2 shows that for \( s = 1 \), for vehicles entering the section after \( t_r \) the predicted VHT is 1.78 veh-hr/mile, and the actual VHT is 2.17 veh-hr/mile, an underprediction of 18%. With increased sensor density, the percent error in underpredicted VHT increases to a maximum of 32% underprediction for \( s = 0.10 \).

When viewing Figure 12 and Table 2 together for transition CE only, it can be seen that when \( s = 1 \), the total error is a 4% overprediction in VHT. For decreasing values of \( s \) when adding numerically the total effect is underprediction, up to 30% with \( s = 0.10 \). If the absolute values of the errors are added, the effect for \( s = 1 \) is 17% error (underprediction), increasing to 30% for \( s = 0.1 \). For \( s = 0.5 \), the total error is 15% and the absolute error is 20%.

Table 2 also shows the results of adding the effects of travel time estimation errors during transitions AC + CE (as shown in Figure 3D). It is possible to consider just the impact of VHT underprediction during the transitions. The additive effects of only underpredicted VMT (data from transitions AC + CE in Table 2) results in errors between 20–29%. For overprediction only the errors range between \(-14\%\) and \(-28\%\).

Table 2 also shows the additive effects of the underprediction and overprediction that occurs in transitions AC and CE. As shown for \( \ell = s = 1 \), the aggregate effect is a \(-0.44\%\) VHT error (very close to zero in the aggregate). The error remains about the same for \( s = 0.10 \). This is also shown graphically in Figure 13. As before, a \( 3\times \) penalty is applied to the underprediction error before it is added to the overprediction error, resulting in a net 18% underprediction for \( s = 1 \) mile, and a 28% underprediction for \( s = 0.1 \) mile. This is also shown in Figure 13. The total error

\[
VHT_{\text{pred}}^{u} = q_{E} \frac{\ell}{v_{f}} \left( \frac{2\ell - s}{2v_{CE}} \right) \left( \frac{\ell}{v_{f}} + \frac{\ell(v_{f} - v_{CE}) + \frac{1}{2}s(v_{c} - v_{f})}{v_{c}(v_{f} - v_{CE})} \right) \]  

(25)

The actual VHT for vehicles in transition CE whose travel time was underpredicted is:

\[
VHT_{\text{act}}^{u} = q_{E} \frac{\ell}{2} \left( \frac{2\ell - s}{2v_{CE}} - \frac{\ell}{v_{f}} + \frac{\ell(v_{f} - v_{CE}) + \frac{1}{2}s(v_{c} - v_{f})}{v_{c}(v_{f} - v_{CE})} \right) \]  

(26)
applying the sum of the absolute value of the underprediction and overprediction error is also shown in Table 2 (7–15% error) and in Figure 13.

**CHANGING DETECTOR LOCATIONS**

The results presented so far are dependent on the assumption that a detector is located in the middle of the link (hence the “midpoint” method). We investigate here what would be different if the detectors were located at the upstream and downstream ends of the link. This change
would not be effected by actually moving the detectors, but rather redefining the endpoints of the
link. Rather than repeat the analysis of the entire paper, we will repeat here just that part
Corresponding to the AC transition. It should be clear to the interested reader at that point how to
repeat the analysis for other shock transitions, as well as for other detector locations or
extrapolation methods not included in this paper.

Figure 14 shows transition AC defined so that single detectors are located at the upstream
and downstream ends of the link. This is in contrast with the situation shown in Figure 4. Here
we will repeat the analysis that began with equation (2), for travel time underestimation,
followed by overestimation. We still focus on the vehicle labeled $j_2$, which is the last vehicle to
enter the section without the benefit of information from the sensor. In this case, $j_2$ travels a
distance:

$$x_{AC} = \frac{v_f \ell}{v_f - v_{AC}}$$

(27)

at speed $v_f$ and the remainder of the section at speed $v_c$. The travel time $z$ for vehicle $j_2$ is:

$$z = \frac{v_f \ell}{v_f - v_{AC}} + \frac{v_f \ell}{v_f - v_{AC}} + \frac{v_f \ell - v_{AC} \ell - v_f \ell}{v_f - v_{AC}}$$

(28)

$$= \frac{v_c \ell}{v_c (v_f - v_{AC})} - \frac{v_c \ell (v_c - v_{AC})}{v_c (v_f - v_{AC})}$$

Because a sensor is located at the upstream end, there is no lag ($\alpha = 0$) and the free-flow travel
time is as before. The maximum prediction error $u_{max}$ is therefore:

$$u_{max} = z - \ell_f = \frac{\ell (v_c - v_{AC})}{v_c (v_f - v_{AC})} - \frac{\ell}{v_f}$$

(29)

Figure 14 Transition example during transition AC with detector at downstream end.
The time span over which underpredicted vehicles enter the link is the free-flow travel time plus the lag, which is zero. Again, vehicles are counted using the upstream boundary as a reference, and at this location the flow is \( q_d \); hence the number of vehicles in this condition is:

\[
 n_{ACs} = q_d \left( \frac{\ell}{v_f} \right) 
\]  

(30)

Each of these vehicles has predicted travel time from the VMS equal to the free-flow travel time, so the total VHT per hour predicted for those vehicles that ultimately end up with underpredicted travel times is:

\[
 VHT^u_{pred} = q_d \left( \frac{\ell}{v_f} \right) \left( \frac{\ell}{v_f} \right) = q_d \frac{\ell^2}{v_f^2} 
\]  

(31)

The actual travel times range from the free-flow travel time to the worst-case scenario given in (28); hence the actual VHT per hour is given by:

\[
 VHT^u_{act} = \frac{q_d \ell}{2v_f} \left( \frac{\ell}{v_f} + \frac{\ell (v_c - v_{AC})}{v_c (v_f - v_{AC})} \right) 
\]  

(32)

For vehicles in this situation whose travel time is overpredicted, the time period over which they cross the upstream end of the link is:

\[
 t_{ACo} = -\frac{\ell}{v_{AC}} 
\]

The flow at this location is \( q_d \), so the number of vehicles in this condition is:

\[
 n_{ACo} = -\frac{q_d \ell}{v_{AC}} 
\]  

(33)

These vehicles expect to travel the link at speed \( v_c \), so the total predicted VHT per hour is:

\[
 VHT^o_{pred} = -\frac{q_d \ell^2}{v_{AC} v_c} 
\]  

(34)

The actual VHT per hour is:

\[
 VHT^o_{act} = -\frac{q_d \ell}{2v_{AC}} \left( \frac{\ell}{v_c} + \frac{\ell (v_c - v_{AC})}{v_c (v_f - v_{AC})} \right) 
\]  

(35)

Table 3 shows how these new results compare with those from the midpoint method, for a single detector and for transition AC only. Not surprisingly, situating the detector on the downstream end

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Segment travel time performance measures during regime AC for a downstream detector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime AC</td>
<td>Underprediction</td>
</tr>
<tr>
<td>Detector Location</td>
<td>( u_{max} )</td>
</tr>
<tr>
<td>Midpoint</td>
<td>0.56</td>
</tr>
<tr>
<td>Endpoint</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: Positive error percentages indicate underprediction, while negative error percentages indicate overprediction.
end allows the shock information to be known much earlier, therefore, there is a lower tendency to underpredict travel times but a greater tendency to overpredict. Other detector locations or extrapolation methods (Coifman, 2002) could also be tested for specific situations, using the same process shown here but with different details pertaining to detector location.

**CONCLUSIONS**

This paper has attempted to begin to address the question: “how much detection do you need?” in the context of accurate estimation of freeway travel time using the midpoint method for traveler information applications via upstream roadside VMS. There is no way that one paper can address every possible issue with regard to travel time estimation. Also, it is understood that detection decisions are not made in isolation from other issues, and in fact, sensors are usually placed to enable operation of ramp metering (e.g. Portland) and traffic monitoring (e.g., counting, speed maps, and incident detection). Freeway travel time estimation is often a useful side benefit that can be leveraged from an existing sensor network. It is also possible that in the future some combination of fixed infrastructure based sensors and vehicle based sensing (e.g. AVL) may provide additional answers and improvements. However, this analysis has taken the question of sensor density in some degree of isolation which has resulted in some helpful outcomes. An issue that has been left for further research is the question of where to optimally place sensors, beyond simply a question of spacing. The use of other travel time algorithms beyond the midpoint method should also be explored further. The optimal placement of sensors in relation to known bottlenecks, and high incident locations will be examined in the future.

Even were these results to be applied directly, the method does not culminate in a single “optimal” answer because tradeoffs exist in the space blending drivers’ experiences and expectations. Research might be undertaken to reveal some preference structure and illuminate unknown parameters, and it might also be necessary to apply some level of professional judgment and/or policy imposition to the decision. The proper balance between under- and overprediction, for example, is not known, and may not be the same for all drivers. Ultimately, the goal of this paper has been to highlight these issues and to put the quantitative aspects of the problem on sound footing.

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